Lecture 12

Part 2: AO System Optimization

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Optimization of AO systems

- If you are designing a new AO system:
  - How many actuators?
  - What kind of deformable mirror?
  - What type of wavefront sensor?
  - How fast a sampling rate and control bandwidth (peak capacity)?
- If you are using an existing AO system:
  - How long should you integrate on the wavefront sensor? How fast should the control loop run?
  - Is it better to use a bright guide star far away, or a dimmer star close by?
  - What wavelength should you use to observe?

Issues for designer of astronomical AO systems

- **Performance goals:**
  - Sky coverage fraction, observing wavelength, degree of image compensation needed for science program
- **Parameters of the observatory:**
  - Turbulence characteristics (mean and variability), telescope and instrument optical errors, availability of laser guide stars
- **AO parameters chosen in the design phase:**
  - Number of actuators, wavefront sensor type and sample rate, servo bandwidth, laser characteristics
- **AO parameters adjusted by user:** integration time on wavefront sensor, wavelength, guide star mag. & offset

Example: Keck Observatory AO "Blue Book"

- Made scientific case for Keck adaptive optics system
- Laid out the technical tradeoffs
- Presented performance estimates for realistic conditions
- First draft of design requirements

The basis for obtaining funding commitment from the user community and observatory

What is in the Keck AO Blue Book?

- **Chapter titles:**

  1. Introduction
  2. Scientific Rationale and Objectives
  3. Characteristics of Sky, Atmosphere, and Telescope
  4. Limitations and Expected Performance of Adaptive Optics at Keck
  5. Facility Design Requirements

- **Appendices: Technical details and overall error budget**

Other telescope projects have similar "Books"

- Keck Telescope (10 m):
  - Had a "Blue Book" for the telescope concept itself
- Thirty Meter Telescope:
  - Series of design documents: Detailed Science Case, Science Based Requirements Document, Observatory Requirements Document, Operations Requirements Document, etc.

These documents are the kick-off point for work on the "Preliminary Design"

First, look at individual terms in error budget one by one

- Error budget terms
  - Fitting error
  - WFS measurement error
  - Anisoplanatism
  - Temporal error

- Figures of merit
  - Strehl ratio
  - FWHM
  - Encircled energy
  - Strehl ratio

Fitting error: dependence of Strehl on λ and DM degrees of freedom

Deformable mirror fitting error only

$$S = \exp\left(-\sigma_\phi^2\right) = \exp\left[-0.28\left(\frac{d}{r_0}\right)^{5/3}\right]$$

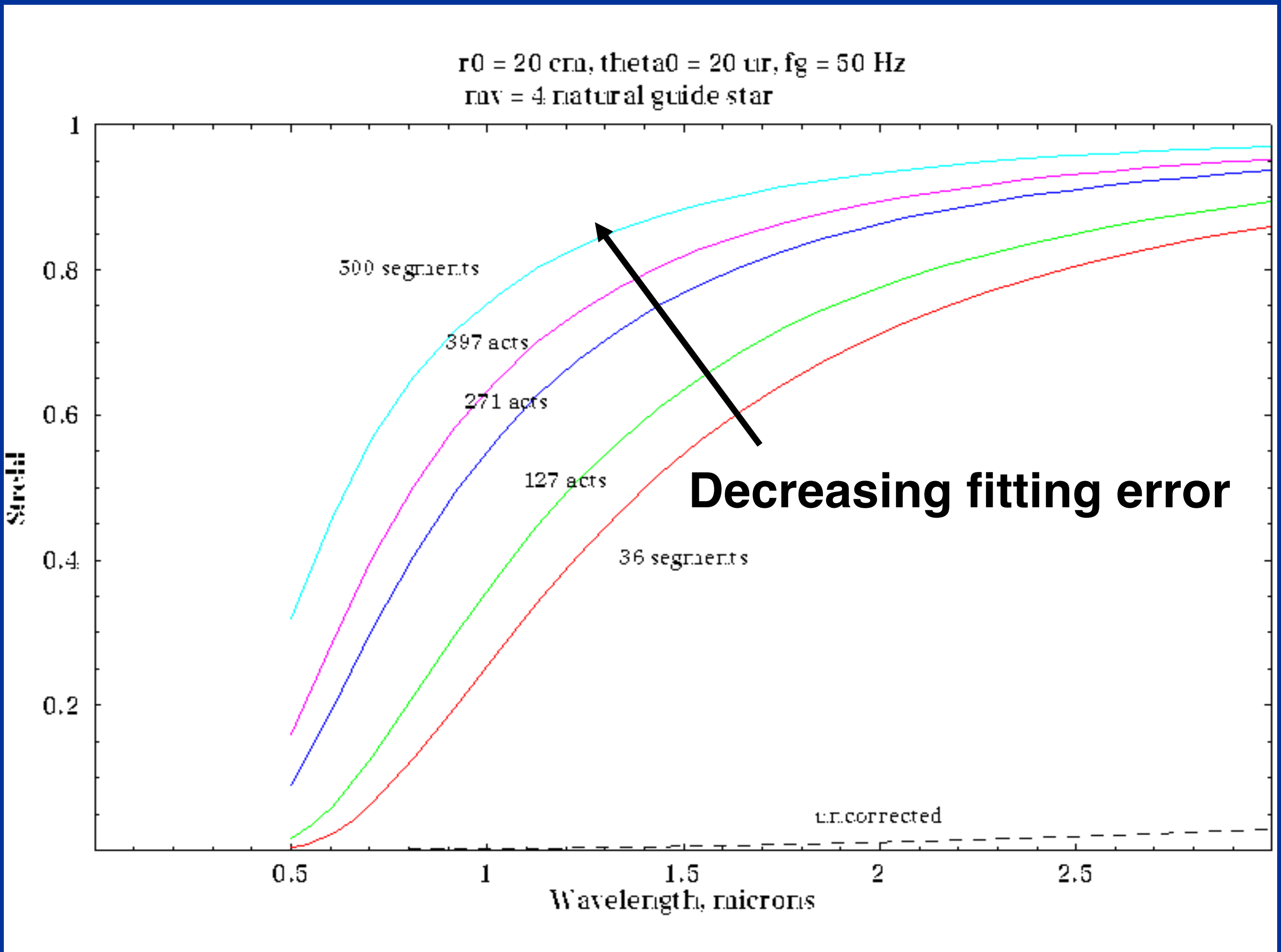
$$r_0(\lambda) = r_0(\lambda = 0.5\mu m)\left(\frac{\lambda}{0.5\mu m}\right)^{6/5}$$

$$S = \exp\left[-0.28\left(\frac{d}{r_0(\lambda = 0.5\mu m)}\right)^{5/3}\left(\frac{0.5\mu m}{\lambda}\right)^2\right]$$

- Assume very bright natural guide star
- No meas't error or anisoplan-atism or band-width error

Strehl increases for smaller subapertures and shorter observing wavelengths

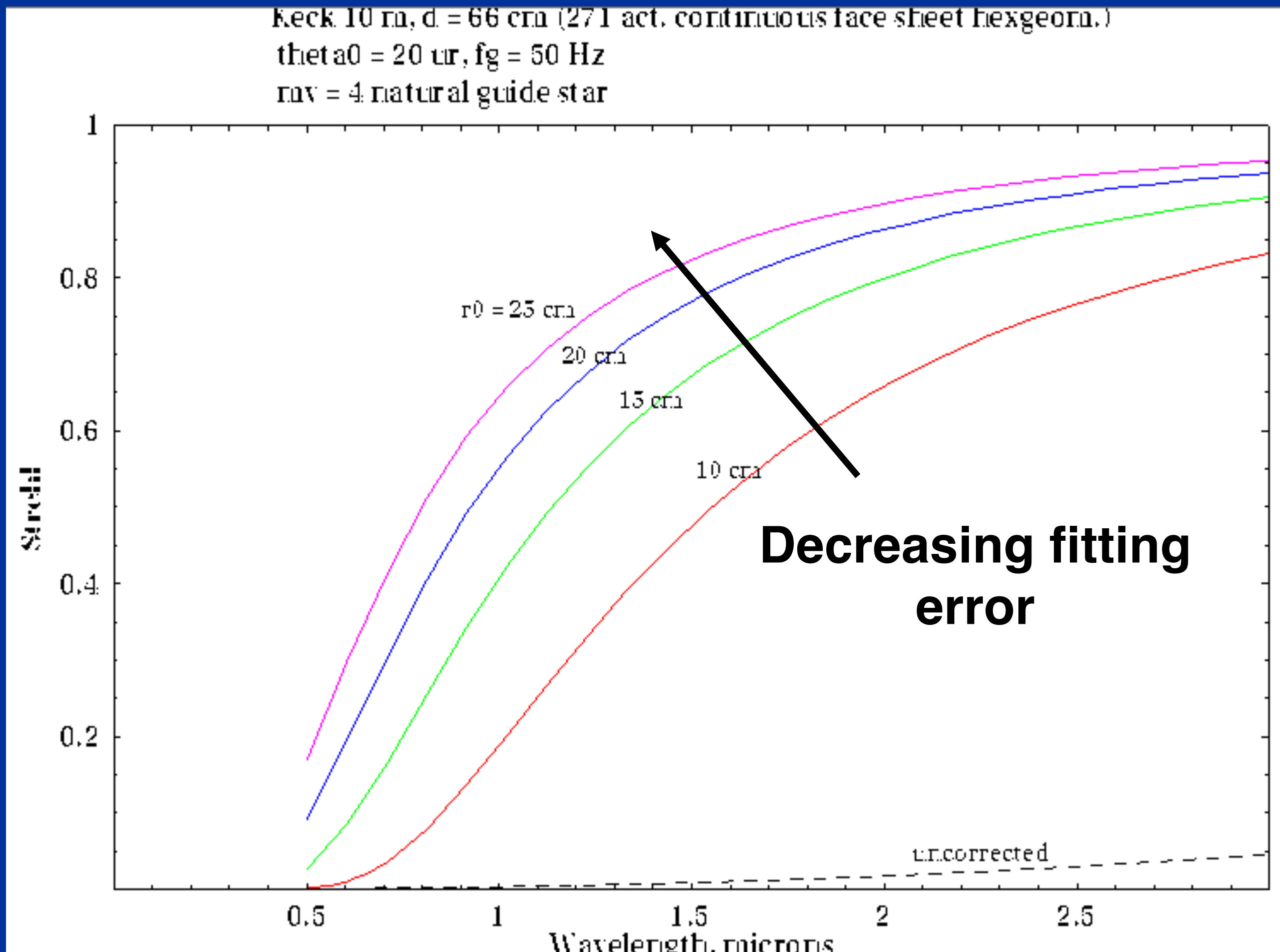
Strehl increases for smaller subapertures and longer observing wavelengths



- Assume very bright natural guide star
- No meas't error or anisoplan-atism or band-width error

Deformable mirror fitting error only

Strehl increases for longer λ and better seeing (larger r_0)



- Assume very bright natural guide star
- No meas't error or anisoplan-atism or band-width error

Deformable mirror fitting error only

Wavefront sensor measurement error: Strehl vs λ and guide star magnitude

Assumes no DM fitting error or other error terms

$$\sigma^2_{S-H} \approx \left(\frac{6.3}{SNR}\right)^2$$

$$S = \exp\left(-\sigma^2_{S-H}\right) = \exp\left[-\left(\frac{6.3}{SNR}\right)^2\right]$$

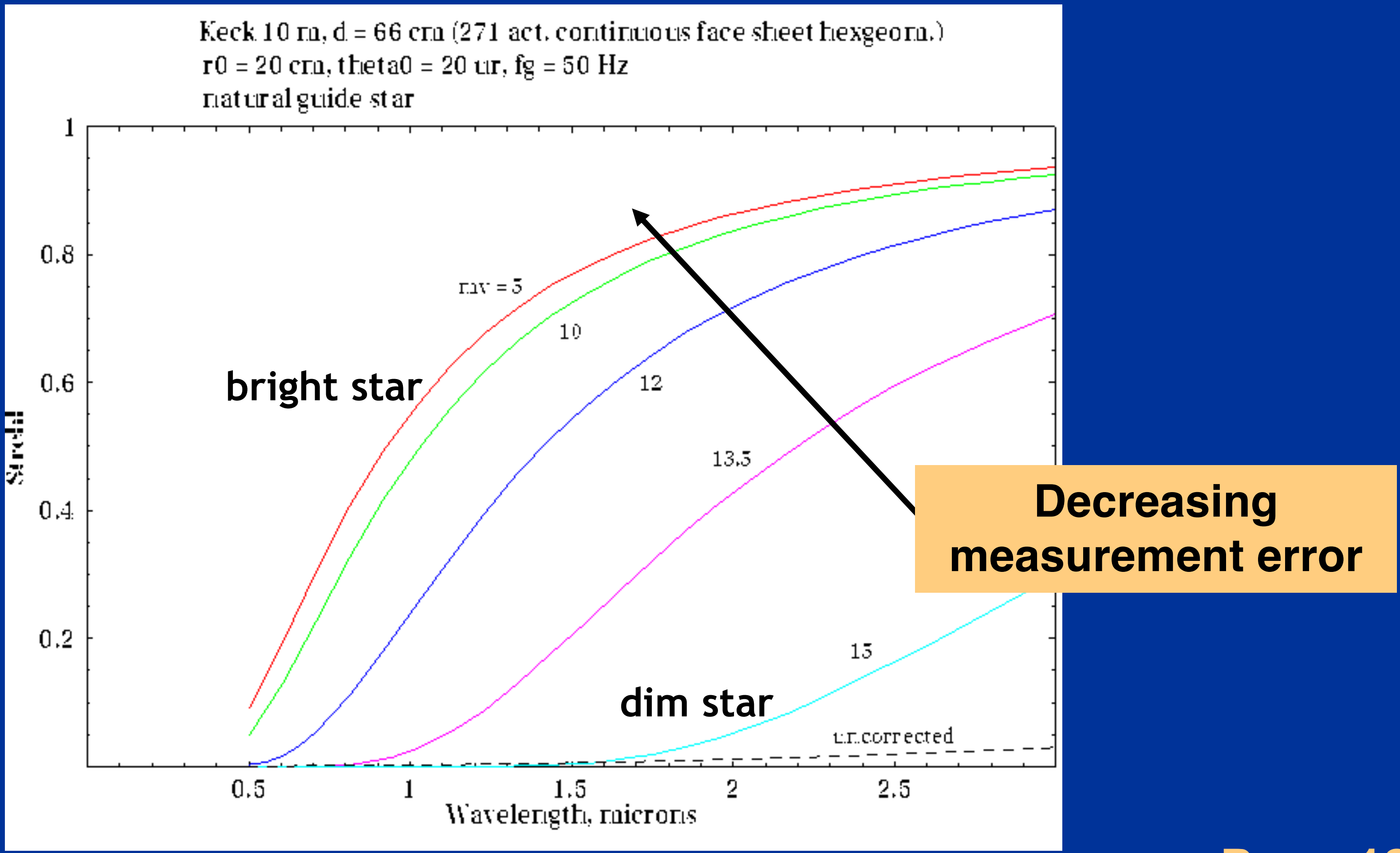
SNR increases as flux from guide star increases

Strehl increases for brighter guide stars

But: SNR will decrease as you use more and more subapertures, because each one will gather less light

Strehl increases for brighter guide stars

Assumes no DM fitting error or other error terms



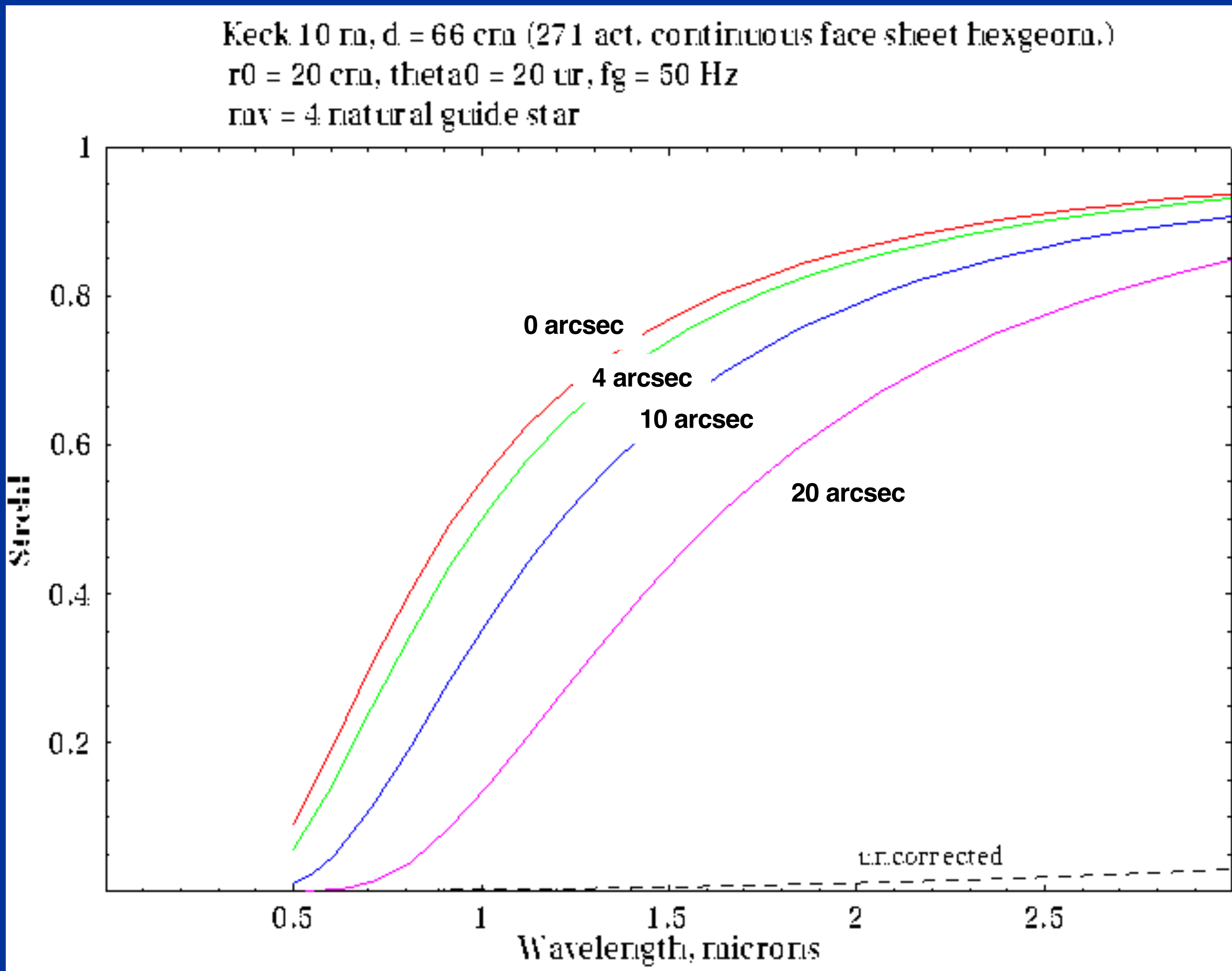
Strehl vs λ and guide star angular separation (anisoplanatism)

$$S = \exp\left[-\sigma_{iso}^2\right] = \exp\left[-\left(\frac{\theta}{\theta_0}\right)^{5/3}\right], \qquad \theta_0 = \frac{r_0}{\bar{h}} \propto \lambda^{6/5}$$

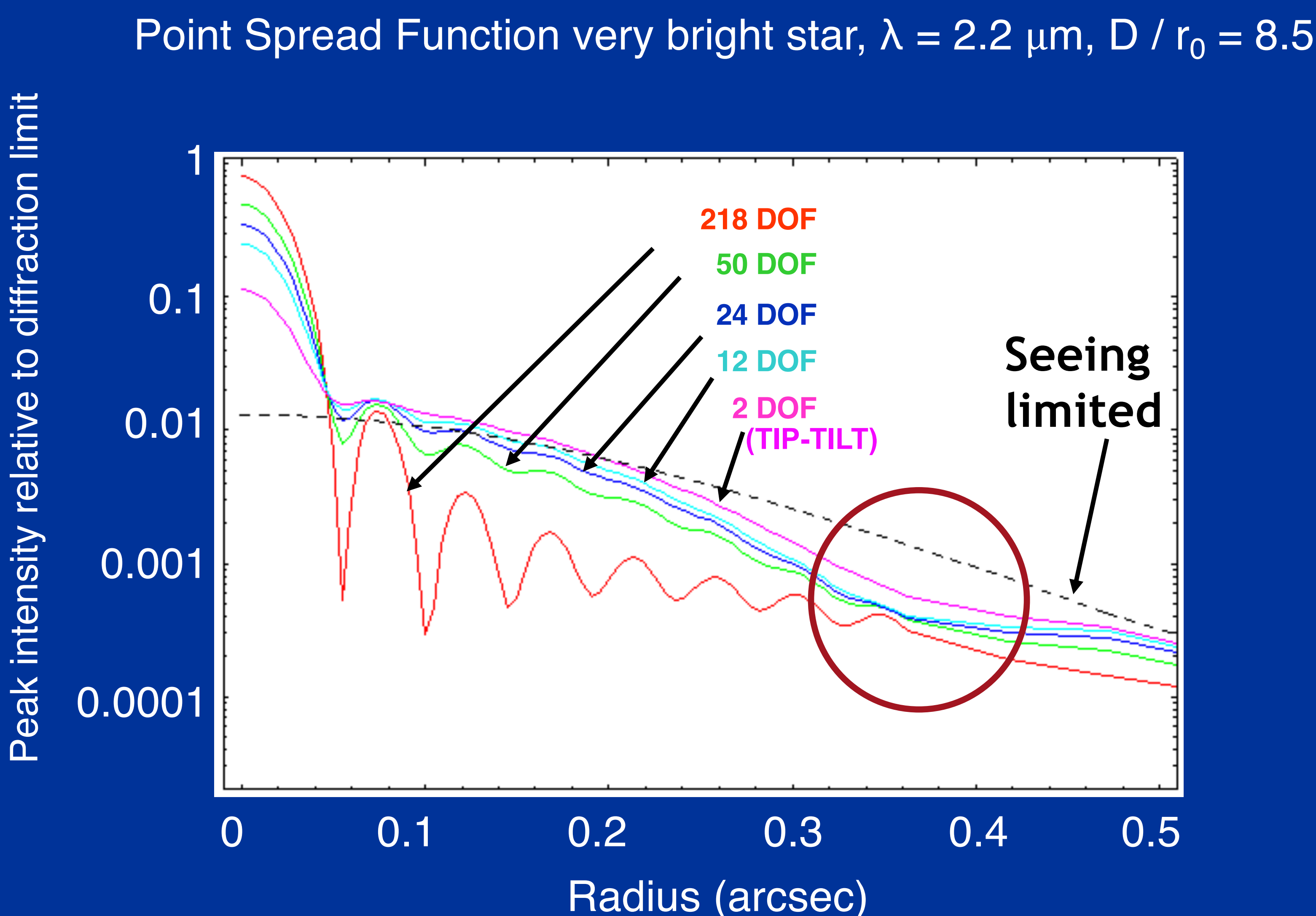
$$S = \exp\left[-\left(\frac{\theta}{\theta_0(0.5\mu m)}\right)^{5/3}\left(\frac{0.5\mu m}{\lambda}\right)^2\right]$$

Strehl increases for smaller angular offsets and longer observing wavelengths

Strehl increases for smaller angular offsets and longer observing wavelengths



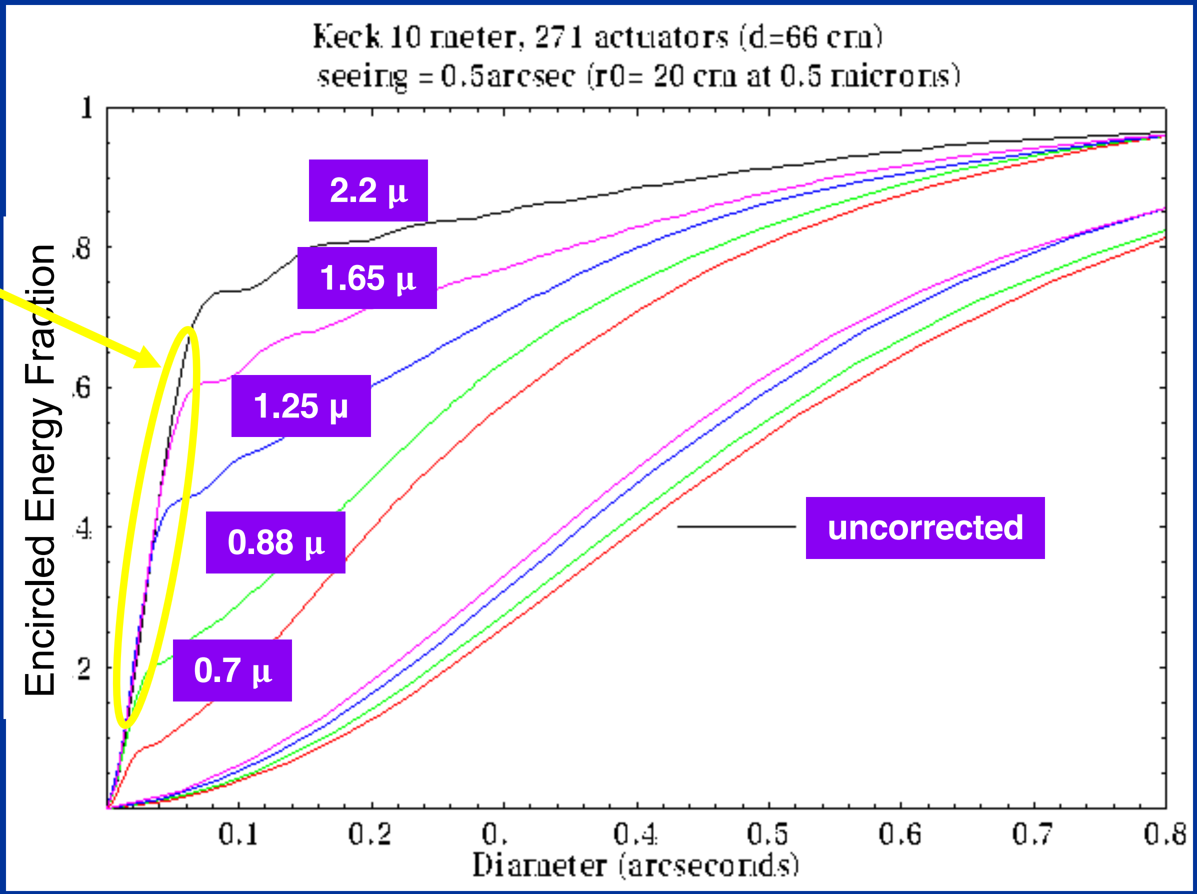
PSF with bright guide star: more degrees of freedom ⇒ more energy in core



What matters for spectroscopy is "Encircled Energy"

Fraction of light encircled within diameter of xx arc sec

Diffraction limited



Overall system optimization

- Concept of error budget
  - Independent contributions to wavefront error from many sources
- Minimize overall error with respect to a parameter such as integration time or subaperture size

Error model: mean square wavefront error is sum of squares of component errors

- Mean square error in wavefront phase

$$\sigma_p^2 = \sigma_{Meas}^2 + \sigma_{BW}^2 + \sigma_{DM}^2 + \sigma_{iso}^2 + \sigma_{T-T}^2 + \ldots..$$

Meas't Timelag Fitting Isoplan. Tip-tilt

$$\sigma_{Meast}^2 = \sigma_{S-H}^2 \approx \left[\frac{3.5\theta_b}{SNR} \frac{d}{\lambda} \right]^2$$

Signal to Noise Ratio for a fast CCD detector

$$SNR = \frac{Flux \times T_{\mathrm{int}}}{Noise} = \frac{Flux \times T_{\mathrm{int}}}{\left[\sigma^2_{PhotonNoise} + \sigma^2_{SkyBkgnd} + \sigma^2_{DarkCurrent} + \sigma^2_{\mathrm{ReadNoise}}\right]^{1/2}}$$

- *Flux* is the average photon flux (detected photons/sec)
- T_{int} is the integration time of the measurement,
- *Sky background* is due to OH lines and thermal emission
- *Dark current* is detector noise per sec even in absence of light (usually due to thermal effects)
- *Read noise* is due to the on-chip amplifier that reads out the charge after each exposure

Short readout times needed for wavefront sensor ⇒ read noise is usually dominant

- *Read-noise dominated*: read noise >> all other noise sources

- In this case SNR is

$$SNR_{RN} = \frac{Flux \times T_{\text{int}}}{\left[R^2 n_{pix}\right]^{1/2}} = \frac{Flux \times T_{\text{int}}}{R\sqrt{n_{pix}}}$$

where T_{int} is the integration time, n_{pix} is the number of pixels in a subaperture, *R* is the read noise/px/frame

Now, back to calculating measurement error for Shack-Hartmann sensor

$$\sigma^2_{S-H} \cong \left[\frac{\pi}{4\sqrt{2}} \frac{1}{SNR} \vartheta_b \frac{2\pi d}{\lambda} \right]^2 \text{rad}^2$$

- Assume the WFS is read-noise limited. Then

$$SNR_{S-H} = \frac{Flux \times T_{\text{int}}}{R\sqrt{n_{pix}}} \quad \text{and}$$

$$\sigma^2_{S-H} = \left[\frac{3.5\theta_b}{SNR} \frac{d}{\lambda} \right]^2 = \left[3.5\theta_b \frac{d}{\lambda} \right]^2 \frac{R^2 n_{pix}}{\left(Flux \times T_{\text{int}}\right)^2}$$

Error model: mean square wavefront error is sum of squares of component errors

$$\sigma_p^2 = \sigma_{Meast}^2 + \sigma_{BW}^2 + \sigma_{DM}^2 + \sigma_{iso}^2 + ...$$

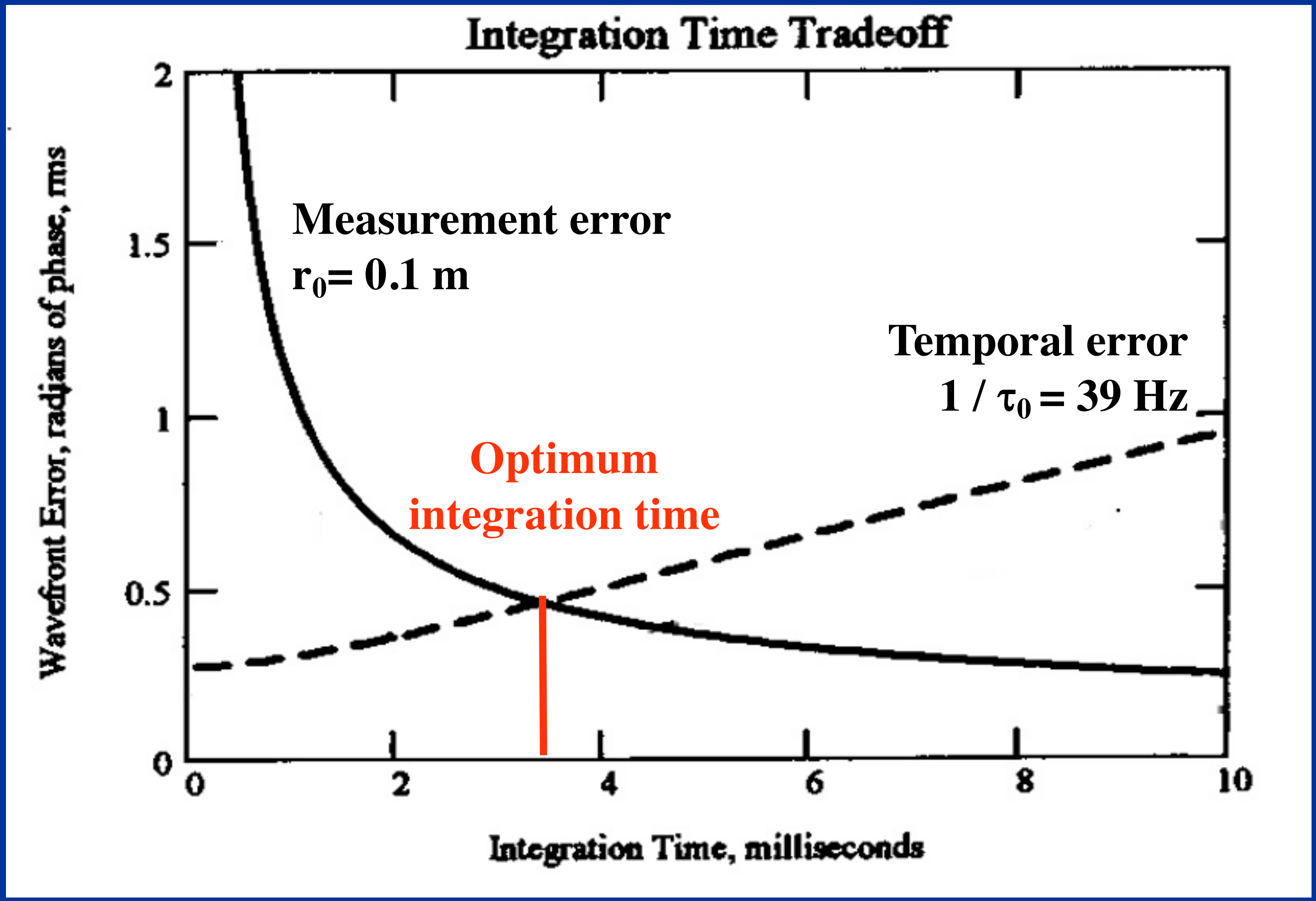
$$\sigma_p^2 = \left[3.5\theta_b \frac{d}{\lambda} \right]^2 \frac{R^2 n_{pix}}{\left(Flux \times T_{\text{int}}\right)^2} + \left(\frac{T_{control}}{\tau_0} \right)^{5/3} + \mu \left(\frac{d}{r_0} \right)^{5/3} + \left(\frac{\theta}{\theta_0} \right)^{5/3} + ...$$

Flux in a subaperture will increase with subap. area ***d*²**

$T_{control}$ is the closed-loop control timescale, typically ~ 10 times the integration time T_{int} (control loop gain isn’t unity, so must sample many times in order to converge)

Integration time trades temporal error against measurement error

$$\sigma_p^2 \approx \left[3.5\theta_b \frac{d}{\lambda} \right]^2 \frac{R^2 n_{pix}}{(Flux \times T_{\text{int}})^2} + \left(\frac{10 T_{\text{int}}}{\tau_0} \right)^{5/3}$$



From Hardy, Fig. 9.23

First exercise in optimization: Choose optimum integration time

- Minimize the sum of read-noise and temporal errors by finding optimal integration time

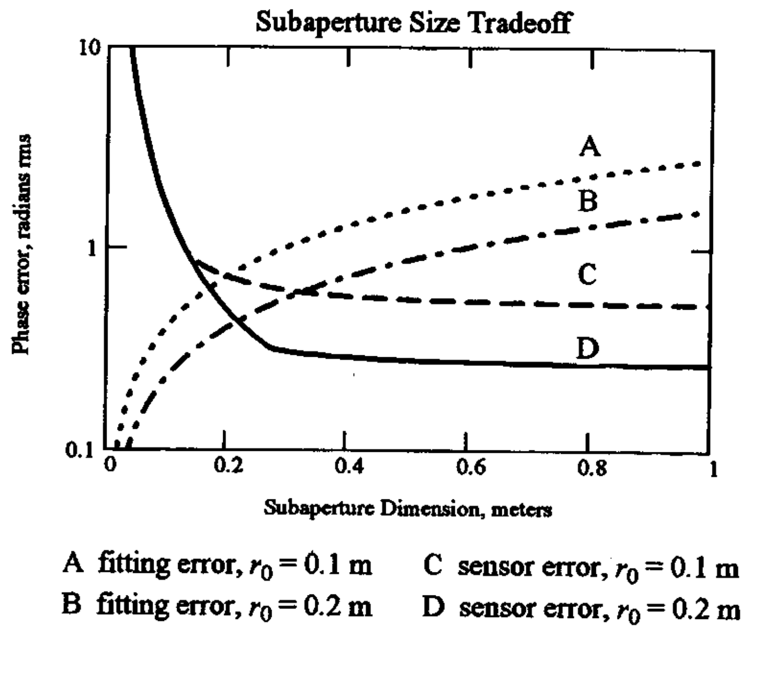
$$\sigma_p^2 = \left[3.5\theta_b \frac{d}{\lambda}\right]^2 \frac{R^2 n_{pix}}{\left(Flux \times T_{\text{int}}\right)^2} + \left(\frac{10 T_{\text{int}}}{\tau_0}\right)^{5/3}$$

$$\frac{d\sigma_p^2}{dT_{\text{int}}} = 0 = -2\left[3.5\theta_b \frac{d}{\lambda}\right]^2 \frac{R^2 n_{pix}}{\left(Flux^2 T_{\text{int}}^3\right)} + \frac{5}{3}\left(\frac{10}{\tau_0}\right)^{5/3} T_{\text{int}}^{2/3}$$

$$T_{\text{int}}^{opt} = \left[0.32\tau_0^{5/3}\left(\frac{\theta_b d}{\lambda}\right)^2 \frac{R^2 n_{pix}}{\left(Flux^2\right)}\right]^{3/11}$$

- Sanity check: optimum T_{int} larger for long τ_0, larger read noise *R*, and lower photon $Flux$

Similarly, subaperture size d trades fitting error against measurement error



Hardy, Figure 9.25

$$\sigma_\phi^2 = \left[3.5\theta_b \frac{d}{\lambda}\right]^2 \frac{R^2 n_{pix}}{(Flux \times T_{\text{int}})^2} + \mu\left(\frac{d}{r_0}\right)^{5/3}$$

$Flux = I \times area = I \times d^2$ where intensity I = photons/(sec cm^2)

$$\sigma_\phi^2 = \left[3.5\theta_b \frac{d}{\lambda}\right]^2 \frac{R^2 n_{pix}}{(I \times d^2 \times T_{\text{int}})^2} + \mu\left(\frac{d}{r_0}\right)^{5/3}$$

$$\sigma_\phi^2 = \left[3.5\theta_b \frac{1}{\lambda}\right]^2 \frac{R^2 n_{pix}}{(I \times T_{\text{int}})^2 d^2} + \mu\left(\frac{d}{r_0}\right)^{5/3}$$

- Smaller d: better fitting error, worse measurement error

Solve for optimum subaperture size d

$$\sigma_\phi^2 = \left[3.5\theta_b \frac{d}{\lambda}\right]^2 \frac{R^2 n_{pix}}{\left(Flux \times T_{\text{int}}\right)^2} + \mu\left(\frac{d}{r_0}\right)^{5/3}$$

$Flux = I \times area = I \times d^2$ where intensity I = detected photons/(sec cm^2)

$$\sigma_\phi^2 = \left[3.5\theta_b \frac{d}{\lambda}\right]^2 \frac{R^2 n_{pix}}{\left(I \times d^2 \times T_{\text{int}}\right)^2} + \mu\left(\frac{d}{r_0}\right)^{5/3} = \left[3.5\theta_b \frac{1}{\lambda}\right]^2 \frac{R^2 n_{pix}}{\left(I \times T_{\text{int}}\right)^2 d^2} + \mu\left(\frac{d}{r_0}\right)^{5/3}$$

Set derivative of σ_ϕ^2 with respect to subaperture size d equal to zero:

$$d_{opt} = \left\{\frac{6}{5}\frac{r_0^{5/3}}{\mu}\left[\frac{3.5\theta_b}{\lambda}\right]^2 \frac{R^2 n_{pix}}{(I \times T_{\text{int}})^2}\right\}^{3/11}$$

d_{opt} is larger if r_0, read noise, and n_{pix} are larger, and if T_{int} and I are smaller

LGS (10th mag TT star) Case	Wavefront Error (nm)
Atmospheric Fitting Error	110
Bandwidth Error	146
High-order Measurement Error	150
LGS Focal Anisoplanatism Error	208
Uncorrectable Static Telescope Abs	66
Dyn WFS Zero-point Calib Error	80
Residual Na Layer Focus Change	36
High-Order Aliasing Error	37
Uncorrectable AO System Aberrations	30
Uncorrectable Instrum Aberrations	110
Angular Anisoplanatism Error	24
Tilt Measurement Error (one-axis)	11
Tilt Bandwidth Error (one-axis)	18
Residual Tel Pointing Jitter (1-axis)	96
Total Wavefront Error (nm) =	370
Strehl at K-band =	0.3
Assumptions	
Zenith angle (deg)	10
Guide star magnitude	10
HO WFS Rate (Hz)	500
TT Rate (Hz)	1000
Laser power (W)	16
WFS camera	CCD39
Subaperture diameter (m)	0.5625
Science Instrument	NIRC2
Amplitude of vibrations (arcsec)	0.2
d0 (m)	2.41
TT sensor	STRAP APDs

Keck 2 AO error budget example

(bright TT star)

Summary: What can you optimize when?

- Once telescope is built on a particular site, you don't have control over τ_0, θ_0, r_0
- But when you *build* your AO system, you CAN optimize choice of subaperture size d, maximum AO system speed, range of observing wavelengths, sky coverage, etc.
- Even when you are observing with an *existing* AO system, you can optimize:
  - wavelength of observations (changes fitting error)
  - integration time of wavefront sensor T_{int}
  - tip-tilt bandwidth
  - brightness and angular offset of guide star